# Optical implementation of a constant-time multicomparand bit-parallel magnitude-comparison algorithm using wavelength- and polarization-division multiplexing with application to parallel database processing 

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Received May 15, 1998


#### Abstract

We present a word- and bit-parallel magnitude-comparison architecture that permits multiple comparands to be compared with multiple relations in constant time. The proposed magnitude-comparison algorithm uses a novel polarization and wavelength-encoding scheme to achieve a fast, scalable realization. Distinctive features of the proposed architecture include (1) the use of a multiple-wavelength encoding scheme to increase processing parallelism and (2) multiple-comparand word- and bit-parallel comparison with an execution time that is independent of the data or word size. © 1998 Optical Society of America

OCIS codes: 200.2610, 200.4540, 200.4860, 200.4960, 200.4560.


Optical nonnumerical-symbolic processing, with its innate spatial parallelism, three-dimensional interconnection, large storage capacity, and low cross talk, has been well recognized as an excellent candidate for meeting current demands and future trends for very high-speed database processing. ${ }^{1-4}$

Several algorithms have been proposed that optically implement equivalence and magnitude-comparison processing. ${ }^{5-7}$ Magnitude comparison has typically been processed in bit-serial fashion, ${ }^{4,8}$ which results in an execution time that is highly dependent on word size. One magnitude-comparison algorithm ${ }^{9}$ succeeds in achieving word and bit parallelism, but it requires that multiple comparands be processed in serial fashion. In this Letter we present an algorithm and architecture that perform magnitude- comparison processing in a multiple-comparand word-parallel (many words compared with many words) and bit-parallel (many bits compared simultaneously) manner.
Binary patterns are represented by spatially distributed orthogonally polarized locations on a twodimensional pixelated grid. We define logical 1 as vertically polarized light and logical 0 as horizontally polarized light. Individual tuples are differentiated from each other by polarization encoding of each to a unique wavelength.

Only the comparison of a single comparand with a single tuple is shown for clarity and graphical simplicity. The output of this system is a register that indicates the relative magnitude of $n$ number of comparands against $m$ number of relational tuples. The system is capable of determining greater than and less than or equal to. To differentiate a lessthan result from an exact match requires further processing. ${ }^{9}$
A block diagram of the magnitude comparison's functional schematic organization is shown in Fig. 1. The comparand array (CA) contains all the tuples to be matched in parallel, and the relational array (RA) contains all the tuples to be matched against. Both
the CA and the RA enter one of two identical rank comparison units (RCU's) and are processed in parallel. The RCU that is responsible for CA processing is termed the comparand rank comparison (CRC) unit, and the RCU responsible for RA processing is called the relational rank comparison (RRC) unit.
Below a single comparand bit pattern ( $1-1-0$ ) is compared with a single relational bit pattern ( $1-1-1$ ). Figure 2 illustrates the hardware implementation of the RCU as configured for comparand comparison. The RCU takes a stimulus light plane and performs an equivalency comparison of the input with a ranked binary lookup table. This operation reduces the input word to a single pixel whose vertical position indicates its absolute magnitude. The process begins with a two-dimensional multiwavelength source array (SA1) in which each row is designed to emit at a separate wavelength. We are only concerned with propagating a single comparand through the system, so only the first row is selected. All bit positions are first reset to the 0 logic level (LP1). Light plane LP1 then passes through an electrically addressable spatial light modulator (SLM1) that is loaded with each of the comparands to be matched, each one on a different row. Each spatial light modulator pixel that holds a value of 1 rotates the polarization of the transmitted light by $90^{\circ}$ to form plane LP2. Light plane LP2


Fig. 1. Magnitude comparison functional schematic organization.


Fig. 2. RCU configured for comparand processing.
impinges upon a cylindrical lens array whose function is to replicate each of the incident rows $n$ times vertically. Every bit position in the CA is projected onto a bit slice (column) of the subsequent lookup table. The lookup table (SLM2) is an electrically addressable spatial light modulator encoded with sequential binary words from 0 to $2^{m}-1$ (one word per row) arranged in ascending order from top to bottom. The number of entries in the binary lookup table, $2^{m}$, is determined by the maximum word length $m$ that the unit is processing. What results (LPS) resembles the xor operation in that there exists an illuminated pixel everywhere that there is a bit-position mismatch between the comparand and the lookup table. This intermediate light plane (LP5) is funneled down to a single column by use of a cylindrical lens (CL3) in preparation for wavelength demultiplexing.
In the general case the various wavelength components of LP6 (each corresponding to a separate comparand) must be spatially separated into individual columns. We can accomplish this by passing the light through a holographic element (HOE1) that deflects the light at an angle that is a function of its wavelength. Since we are only illustrating a single wavelength, this column (LP6) is allowed to deflect to one of the three columns shown (LP7). The pixels in the leftmost column of LP7 represent the equivalence status of the comparand with each of the words in the lookup table. Illuminated pixels correspond to a tuple mismatch, and nonilluminated pixels correspond to a match. In our example we see that LP7 shows us correctly that there is a match between the comparand, $1-1-0$, and the seventh entry in the lookup table, 1-1-0.
The logic of this demultiplexed light plane must now be reversed in preparation for subsequent stages in the magnitude-comparison algorithm. Figure 3(a) illustrates the necessary hardware. Light plane LP7 impinges upon the write side of an optically addressable spatial light modulator (OASLM). A vertically po-
larized light plane produced by the multiwavelength source array (SA2) reflects off of the OASLM's read side. SA2 is designed such that each column emits at a different wavelength. Since we are only comparing a single comparand, only one column is selected to emit. Simultaneously, LP7 impinges upon the write side of SLM3 and encodes a polarization rotation of the reflected light on its read side. All pixels that had their polarizations modulated by SLM3 are blocked by a vertically oriented polarizer (P4). Light plane LP8 is the logic reversal of intermediate plane LP7.

Figure 3(b) illustrates the output light planes from the CRC and RRC units. The CRC unit output (LP8) represents the comparand, $1-1-0$, and similarly the RRC unit output (LP9) represents the relation, 1-1-1.

Figure 4 illustrates the rank thresholding unit. The optical plane from the RRC unit (LP9) encounters a holographic projector element with a specific fan-out function. A pixel in the $i$ th row of the incident light

(a)

Fig. 3. (a) Converting negative logic to positive logic. (b) Sample output light planes from the CRC unit and the RRC unit.


Fig. 4. Rank threshold unit.
plane must be projected to every row $r$ from $r=1$ to $r=i-1$. This function can most easily be realized by creation of an array of discrete subholograms over the surface of the holographic plate. Each subhologram has a specific fan-out function (varying from top to bottom) and is tuned to a specific wavelength of interest (varying left to right). Relating this to our discussion, the illuminated pixel of LP9 occupies row eight. The fan-out function of the holographic element at this location produces a light plane (LP10) in which rows one through seven are illuminated. Meanwhile the optical output from the CRC unit (LP8) impinges upon the write side of an OASLM (SLM4) that rotates the polarization of LP10 that reflects off of its read side. This light plane (LP11) is steered out of the system by a beam splitter (BS1) and mirror (M1) combination. LP11 contains a horizontally polarized light pixel for every CA-RA tuple pair such that the RA tuple exceeds the CA tuple in magnitude. In this example we see from the single horizontally polarized pixel in LP11 that the RA tuple, $1-1-1$, is greater than the CA tuple, $1-1-0$. The pixel resides in the first column of LP11, indicating that it corresponds to the first comparand of the CA. The specific wavelength of this plane determines which tuple in the RA it corresponds to. Since RA tuples are differentiated by wavelength at this stage, adding additional RA tuples would translate into overlapping but independent light planes coexisting in LP11.

LP11 passes through a horizontally oriented polarizer (P5) that allows only the modified pixels to pass. This light plane (LP12) contains the relative magnitude-comparison information between all the tuples in the comparand array and all the tuples in the relational array. The light plane passes through a lens array (CL4 and CL5) that funnels the plane down to a single row. This row then passes through a holographic element that operates in a fashion similar to the one in Fig. 2, except that the light is deflected vertically instead of horizontally.

The decoding of the output register is rather straightforward. The output register's width in pixels is equal to the number of tuples in the comparand array, and the height is equal to the number of tuples in the relational array. For an $m$ by $n$ output register grid, pixel ${ }_{m n}$ is illuminated such that tuple $\mathrm{RA}_{m}$ is greater than tuple $\mathrm{CA}_{n}$.

In this Letter we have proposed a single-pass multicomparand bit- and word-parallel magnitudecomparison algorithm. Optical implementation is made possible by exploitation of the noninteractive behavior of coincident light planes of differing wavelength. Compared with conventional iterative word-parallel magnitude-comparison schemes in which $w$ is the word length and $n$ is the number of comparands to be operated on, this algorithm offers a speed up of $w \times n$. This performance suggests a substantial improvement in database operations such as sorting, which typically use the magnitude-comparison operation repetitively.

This research was supported by National Science Foundation grant MIP-9505872.

## References

1. A. Louri, Appl. Opt. 31, 3241 (1992).
2. P. Mitkas and B. Berra, J. Parallel Distributed Comput. 17, 230 (1993).
3. L. Irakliotis, G. Betzos, and P. Mitkas, in Associative Processors and Processing, A. Krieklis and C. Wheems, eds. (IEEE Computer Society, New York, 1997), p. 155.
4. P. B. Berra, K. H. Brenner, W. T. Chatey, H. J. Caulfield, S. H. Lee, and H. Szu, Appl. Opt. 29, 195 (1990).
5. M. Iwata, J. Tanida, and Y. Ichioka, Appl. Opt. 32, 1987 (1987).
6. P. Guilfoyle, P. Mitkas, and P. Berra, Proc. SPIE 1297, 124 (1990).
7. A. Louri and J. Hatch, Jr., Appl. Opt. 33, 8153 (1994).
8. C. J. Date, An Introduction to Database Systems, 4th ed. (Addison-Wesley, Reading, Mass., 1986), p. 245.
9. A. Louri and J. Hatch, Jr., Opt. Lett. 18, 992 (1993).
